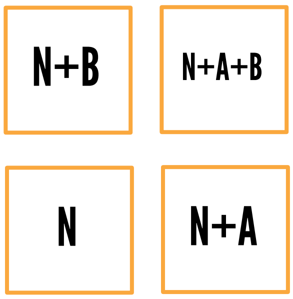
CALCULATOR PUZZLE PROOF

Question

All numbers whose digits go around a rectangle on a calculator (with the number starting on any digit) are multiples of 11. Why is this?

Solution

Convince yourself that the digits in the corners of any rectangle made on the grid are of the form below, where **n**, **a** and **b** are integers.



We need to show every four-digit number made by going around the rectangle, starting on any number and going in either direction, will be a multiple of 11.

Example

If the number starts on a and goes clockwise, it is:

$1000n + 100(n + b) + 10(n + a + b) + n + a$
$= 1111n + 110b + 11a$
$=11(101n + 10b + a)$

The number is a multiple of 11 because the coefficients of **n**, **a** and **b** are all multiples of 11.

But how would this change for other examples?

We can see that because '**n**' is contained in each of the digits in the rectangle, we will always have:

$1000n + 100n + 10n + n = 1111a$

The digits that contain **b** are always adjacent to each other, and this is also true of the two digits that contain **a.**

There are only 4 possible positions that two adjacent digits can go in, and each of these situations will give a coefficient for **b** or **a** that is a multiple of 11. See below.

The coefficient of **b** or **a** could be any of the following:

1000 + 100
= 1100
=11 × 100

100 + 10
=110
=11 × 10
(as for b in the example above)

10 + 1
=11
(as for a in the example above)

1 + 1000
=1001
=11 × 91

As the coefficients of **n**, **b** and **a** are always multiples of 11, therefore the number must always be a multiple of 11.